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Gates & Hydraulic Jumps

1. Context

Consider a rectangular channel with mild slope that conveys a uniform and subcritical flow (see Fig. 1). A sluice gate installed across the channel introduces a local contraction that accelerates the water as it passes beneath the opening. When the gate opening is sufficiently small, the jet formed under the gate becomes supercritical. Because the downstream reach of the channel requires subcritical conditions, the flow must undergo a hydraulic jump to recover the deeper, slower regime. This jump connects section (2), where the flow is supercritical, to section (3), where the flow is subcritical.

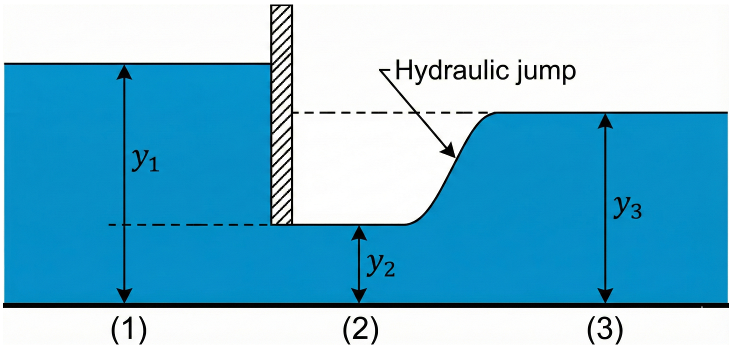

Figure 1: Free hydraulic jump downstream from a sluice gate.

The position and structure of the hydraulic jump are controlled by the balance between the upstream and downstream specific forces, i.e. the momentum balance. When the downstream depth is relatively low, the jump is free to adjust its location and the sequent depth is determined solely by the upstream Froude number. This configuration is known as a free hydraulic jump, and its position may vary along the channel, typically forming downstream of the *vena contracta*. Conversely, if the downstream depth is sufficiently high, it exerts stronger control over the force balance and forces the jump upstream. In this case the roller becomes submerged and the jump is classified as drowned.

The purpose of this document is to investigate how the gate opening determines the limiting condition separating free and drowned jumps. Before analyzing this control mechanism, we review the physical principles that govern hydraulic jumps, with emphasis on the specific force balance and on the way the jump moves when the upstream and downstream specific forces do not match. This background is essential for identifying the gate opening at which the transition between free and drowned behavior occurs.

2. The Hydraulic Jump and Specific Force

A hydraulic jump is a rapidly varied transition from supercritical to subcritical flow in an open channel. It forms when a shallow, fast flow encounters conditions that require a deeper and slower flow. Because supercritical flow cannot adjust its depth gradually, the transition occurs over a short region where strong turbulence dissipates excess energy. The cross sections immediately upstream and downstream of the jump may be treated as one dimensional and nearly uniform, which permits the application of the momentum equation across a control volume that spans the jump.

The forces acting on this control volume are hydrostatic. For a rectangular channel of width b, upstream depth y_1, downstream depth y_2, and mean velocities V_1 and V_2 (see Figure 2), the hydrostatic forces are

$$F_1 = \frac{1}{2}\rho g b\, y_1^2, \qquad F_2 = \frac{1}{2}\rho g b\, y_2^2. \tag{1}$$

The one dimensional steady momentum equation is

$$F_1 - F_2 = \rho Q\,(V_2 - V_1). \tag{2}$$

Using continuity $Q = by_1V_1 = by_2V_2$ and simplifying, one obtains the classical conjugate depth relation expressed through the upstream Froude number

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{gy_1}}, \tag{3}$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8\,\mathrm{Fr}_1^2}\right). \tag{4}$$

This expression defines the sequent depth y_2 that a free hydraulic jump would establish for a given upstream depth y_1.

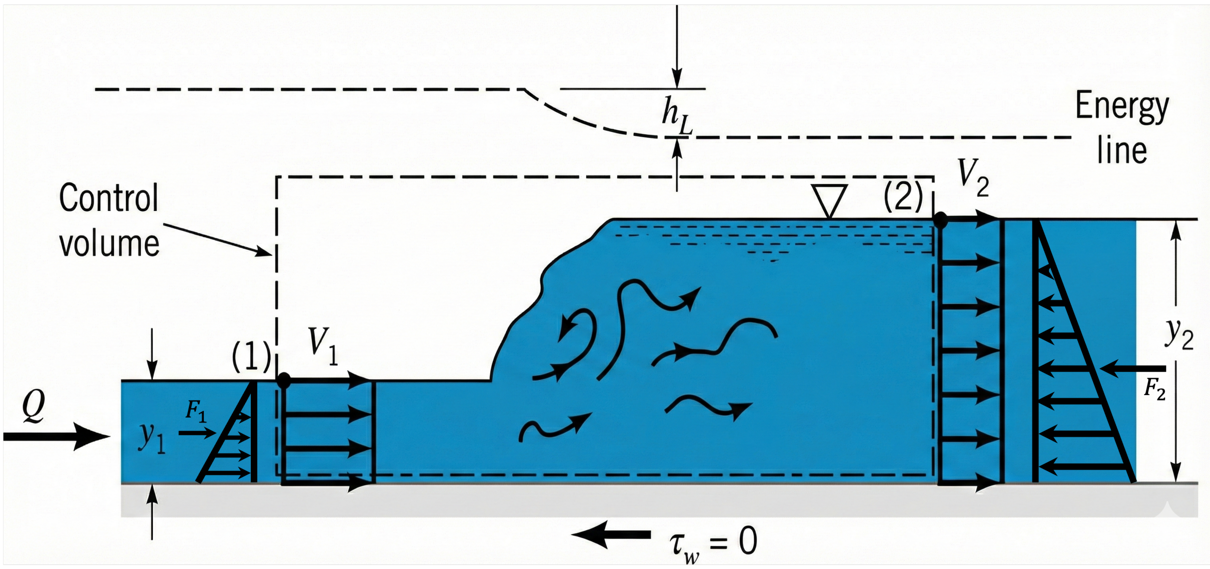

Figure 2: Control volume for a hydraulic jump showing depths, velocities, and hydrostatic forces

Note An alternative form of relation (4) can be written in terms of the downstream Froude number

$$\mathrm{Fr}_2 \;=\; \frac{V_2}{\sqrt{g y_2}}. \tag{5}$$

If equation (4) is algebraically inverted, one obtains

$$\frac{y_1}{y_2} \;=\; \frac{1}{2}\left(-1+\sqrt{1+8\,\mathrm{Fr}_2^2}\right). \tag{6}$$

This form is convenient when the known conditions correspond to the subcritical side of the jump.

Specific Force

The momentum balance can be written in a more compact and physically transparent form using the specific force

$$F_s(y) = \frac{q^2}{g\,y} + \frac{y^2}{2}, \qquad q = \frac{Q}{b}. \tag{7}$$

A stationary hydraulic jump, i.e. a jump that does not move downstream nor upstream, connecting depths y_1 and y_2 (see Figure 3) must satisfy

$$F_s(y_1) = F_s(y_2). \tag{8}$$

For a given discharge per unit width q, the function $F_s(y)$ decreases with depth for supercritical flow and increases for subcritical flow. The turning point corresponds to the critical depth. This monotonicity property on each flow branch is fundamental for determining how a jump moves when the upstream and downstream specific forces differ.

If $F_s(y_1) > F_s(y_2)$, the upstream flow possesses more specific force than the downstream section can balance, so the jump is pushed downstream. If $F_s(y_1) < F_s(y_2)$, the downstream section dominates and the jump moves upstream. A stationary free jump occurs only when the specific forces on both sides match exactly.

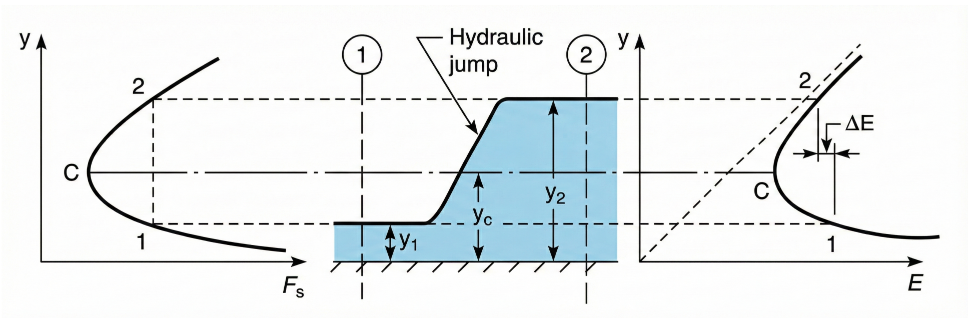

Figure 3: Specific energy and specific force diagrams. Point 1 corresponds to the supercritical flow upstream of the jump, and Point 2 to the subcritical flow downstream.

3. Flow Under a Sluice Gate with Fixed Tailwater Depth

Let's now analyze the hydraulic jump that forms downstream of a sluice gate installed in a rectangular channel that carries a known discharge per unit width q (see Figure 4). The upstream flow is uniform and subcritical. The gate introduces a local contraction that accelerates the flow, producing a supercritical jet immediately downstream of the gate.

For simplicity, we will assume that farther downstream the flow is uniform with a fixed tailwater depth y_{d}. This depth is externally imposed by the channel or by a downstream control structure. Its specific force

$$F_{s,\mathrm{d}} = F_s(y_{\mathrm{d}}) \tag{9}$$

is therefore constant for this analysis.

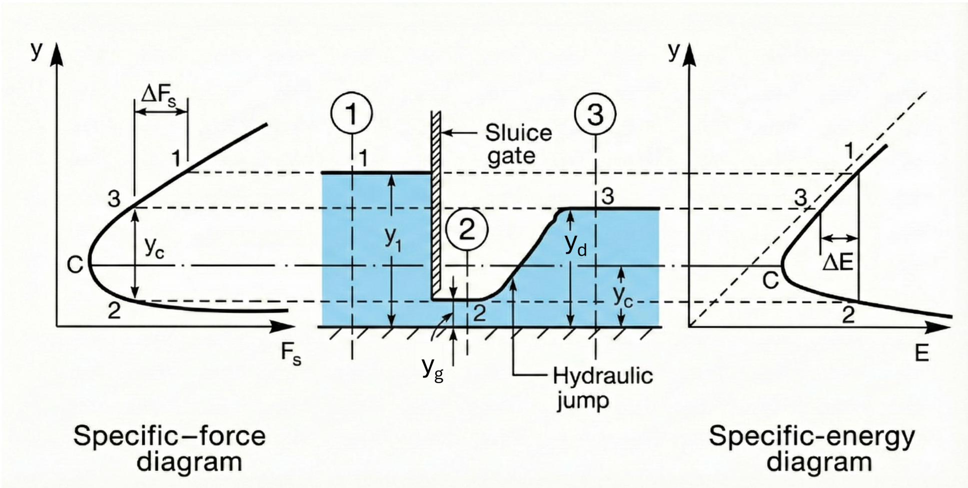

Figure 4: Specific energy and specific force diagrams for a sluice gate discharge. Note the high specific force for small supercritical depths.

Supercritical Flow Beneath the Gate

Let the gate opening be a. Due to contraction at the vena contracta, the effective jet thickness is

$$y_{\mathrm{g}} = C_c\, a, \qquad C_c \approx 0.6. \tag{10}$$

The corresponding velocity and specific force are

$$V_{\mathrm{g}} = \frac{q}{y_{\mathrm{g}}}, \qquad F_{s,\mathrm{g}}(a) = \frac{q^2}{g\, y_{\mathrm{g}}} + \frac{y_{\mathrm{g}}^2}{2} = \frac{q^2}{g\, C_c a} + \frac{(C_c a)^2}{2}. \tag{11}$$

Since y_{g} lies on the supercritical branch, the derivative satisfies

$$\frac{\mathrm{d}F_{s,\mathrm{g}}}{\mathrm{d}a} < 0.$$

Increasing the gate opening reduces the specific force of the jet, while decreasing it produces a thinner, faster, and more energetic jet.

This behavior can be visualized again in Figure 4, which shows the specific energy and specific force diagrams. On the supercritical branch (lower limb of the specific force curve), a decrease in depth (caused by a smaller gate opening) leads to a significant increase in specific force. Conversely, on the subcritical branch (upper limb), the specific force increases with depth. This distinction is crucial: **for the supercritical jet, a smaller a means higher specific force $F_{s,g}$, whereas for the subcritical tailwater, a higher y_d means higher specific force $F_{s,d}$.**

Free or Drowned Hydraulic Jump

As we stated before, the character of the hydraulic jump depends entirely on the comparison between the specific force of the jet under the gate, $F_{s,\mathrm{g}}(a)$, and the specific force of the tailwater, $F_{s,\mathrm{d}}$. This criterion can also be expressed directly in terms of the corresponding depths.

Let $y_\mathrm{g} = C_c a$ be the supercritical depth beneath the gate and let $y_2(y_\mathrm{g})$ denote its conjugate depth,

$$y_2(y_\mathrm{g}) = y_\mathrm{g}\,\frac{1}{2}\left(-1+\sqrt{1+8\,\mathrm{Fr}_\mathrm{g}^2}\right), \qquad \mathrm{Fr}_\mathrm{g} = \frac{V_\mathrm{g}}{\sqrt{g\,y_\mathrm{g}}}.$$

Since $y_2(y_\mathrm{g})$ is the subcritical depth required by a free hydraulic jump attached to the gate, the comparison between y_d and $y_2(y_\mathrm{g})$ provides an equivalent depth based criterion for the type and position of the jump. Let's analyze the three possible cases. **Remember that we are assuming a tailwater depth y_d that is fixed by the channel geometry or by a downstream control structure**.

Case 1: Free jump If we have

$$y_\mathrm{d} < y_2(y_\mathrm{g}) \qquad \Longleftrightarrow \qquad F_{s,\mathrm{g}}(a) > F_{s,\mathrm{d}},$$

the incoming jet possesses more specific force than the tailwater section can balance. The hydrostatic thrust provided by y_d is insufficient to satisfy the momentum equation at the location immediately downstream of the gate, so the jump cannot remain attached to the structure. The imbalance produces a net force directed downstream, and the jump is displaced in that direction.

As the jump moves downstream, the supercritical flow upstream of the jump evolves gradually because the channel has mild slope and the bed shear and pressure variations are small compared with inertial forces. Under these conditions, the flow upstream of the jump can be described by the gradually varied flow equation, and the depth adjusts continuously from the thin supercritical jet just below the gate toward larger values. Since the flow in that region remains supercritical and the depth increases in the downstream direction, the relevant backwater curve is the $M3$ profile: a supercritical profile characterized by a gradual rise in depth over a mild bed slope.

Along this $M3$ profile, the local depth $y(x)$ increases, which reduces the velocity and therefore the upstream specific force. As the jet thickens, the specific force $F_s(y)$ along the supercritical branch decreases and eventually reaches the value $F_{s,\mathrm{d}}$ imposed by the tailwater depth. At that location, the specific force balance can be satisfied and the hydraulic jump becomes stationary. In this configuration, the roller is fully exposed to the atmosphere and the jump behaves as a classical free hydraulic jump.

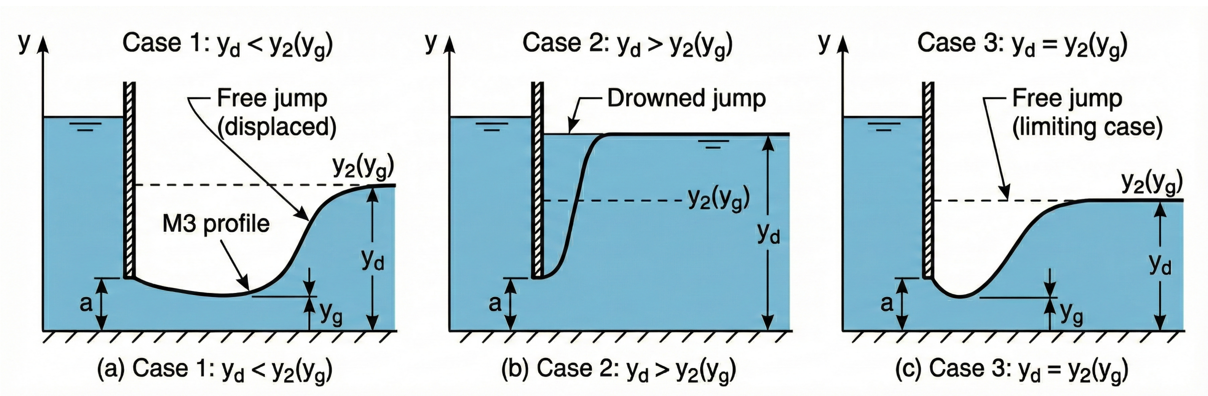

Figure 5: Free, drowned, and limiting hydraulic jump cases. Case 1 produces a free jump far downstream from the gate, Case 2 a drowned jump, and Case 3 is jump that forms immediately downstream from the gate.

Case 2: Drowned jump If we have the contrary case, i.e. the tailwater section provides excess hydrostatic force, we have

$$y_2(y_{\rm g}) < y_{\rm d} \qquad \Longleftrightarrow \qquad F_{s,{\rm g}}(a) < F_{s,{\rm d}},$$

the tailwater section provides excess hydrostatic force. The jump shifts upstream and becomes progressively submerged. The upstream flow deepens and the roller is suppressed, producing a drowned or submerged hydraulic jump.

Case 3: Limiting gate opening The final case is the limiting case, where a stationary free jump forms immediately downstream of the gate:

$$F_{s,{\rm g}}(a_{\rm l\it im}) = F_{s,{\rm d}} \qquad \Longleftrightarrow \qquad y_{\rm d} = y_2(y_{\rm g}).$$

Substituting the expression for $F_{s,{\rm g}}(a)$ gives

$$\frac{q^2}{g\,C_c a_{\rm l\it im}} + \frac{(C_c a_{\rm l\it im})^2}{2} = \frac{q^2}{g\,y_{\rm d}} + \frac{y_{\rm d}^2}{2}. \tag{12}$$

This equation defines the critical opening $a_{\rm l\it im}$ that separates free from drowned hydraulic jumps.

4. Example Application

Consider a 5-m wide rectangular outlet downstream of a control gate. The gate opening is $a = 3.33$ m and the contraction coefficient is $C_c = 0.6$, resulting in a flow depth just downstream of the gate of $y_g = C_c a \approx 2.0$ m. The discharge is $Q = 150$ m^3/s.

For parts 1–3, assume that there are no downstream constraints (free hydraulic jump forms naturally).

We wish to determine:

1. The flow depth downstream of the hydraulic jump.
2. The head loss in the jump.
3. The thrust on the gate.
4. **Effect of High Tailwater**: Now assume a fixed tailwater depth $y_d = 10$ m is imposed downstream. Prove that the jump becomes drowned and calculate the new limiting gate opening required to restore a free hydraulic jump.
5. **Effect of Low Tailwater**: Instead, assume the tailwater depth drops to $y_d = 6$ m. Describe what happens to the jump position and calculate the gate opening required to bring the jump back to the gate.

Solution

First, we calculate the discharge per unit width:

$$q = \frac{Q}{b} = \frac{150}{5} = 30 \text{ m}^2/\text{s}.$$

The velocity and Froude number at the gate (supercritical section) are:

$$V_g = \frac{q}{y_g} = \frac{30}{2} = 15 \text{ m/s},$$

$$\text{Fr}_g = \frac{V_g}{\sqrt{g y_g}} = \frac{15}{\sqrt{9.81 \times 2}} \approx 3.39.$$

1. Downstream depth (y_{seq}) Using the conjugate depth relation:

$$y_{seq} = \frac{y_g}{2}\left(\sqrt{1 + 8\,\text{Fr}_g^2} - 1\right) = \frac{2}{2}\left(\sqrt{1 + 8(11.47)} - 1\right) = 8.63 \text{ m}.$$

2. Head loss (H_L) The specific energy before and after the jump:

$$E_g = y_g + \frac{V_g^2}{2g} = 2 + \frac{15^2}{2 \times 9.81} = 13.47 \text{ m}.$$

$$V_{seq} = \frac{30}{8.63} = 3.48 \text{ m/s} \quad \Rightarrow \quad E_{seq} = 8.63 + \frac{3.48^2}{2 \times 9.81} = 9.25 \text{ m}.$$
$$H_L = E_g - E_{seq} = 13.47 - 9.25 = 4.22 \text{ m}.$$

3. Thrust on the gate (F_{gate}) Assuming no energy loss through the gate, the upstream reservoir depth y_{res} has the same specific energy as the jet ($E \approx y_{res}$ since $V_{res} \approx 0$):

$$y_{res} \approx 13.47 \text{ m}.$$

The thrust is the difference in hydrostatic force plus momentum flux between the reservoir and the jet. This is equivalent to the difference in specific force multiplied by γb, since the specific force is the sum of the hydrostatic and momentum fluxes normalized by the specific weight γ and width b:

$$F_{gate} = \gamma b[F_s(y_{res}) - F_s(y_g)].$$

$$F_s(y_{res}) = \frac{30^2}{9.81 \times 13.47} + \frac{13.47^2}{2} = 6.81 + 90.72 = 97.53 \text{ m}^2.$$
$$F_s(y_g) = \frac{30^2}{9.81 \times 2} + \frac{2^2}{2} = 45.87 + 2 = 47.87 \text{ m}^2.$$
$$F_{gate} = 9810 \times 5 \times (97.53 - 47.87) = 49050 \times 49.66 \approx 2.436 \times 10^6 \text{ N} = 2436 \text{ kN}.$$

4. Effect of Tailwater ($y_d = 10$ m)

Proof of drowning: We compare the specific force of the jet ($y_g = 2$ m) with the specific force of the tailwater ($y_d = 10$ m). From Part 3, we know $F_s(y_g) = 47.87 \text{ m}^2$. For the tailwater:

$$F_s(y_d) = \frac{q^2}{g y_d} + \frac{y_d^2}{2} = \frac{30^2}{9.81 \times 10} + \frac{10^2}{2} = 9.17 + 50 = 59.17 \text{ m}^2.$$

Since $F_s(y_d) > F_s(y_g)$ ($59.17 > 47.87$), the tailwater force dominates, pushing the jump upstream and drowning the gate.

Limiting gate opening: To restore a free jump, we need to increase the specific force of the jet to match the tailwater force. Since F_s increases as depth decreases (on the supercritical branch), we need a smaller opening. We seek $y_{g'}$ such that:

$$\frac{30^2}{9.81\, y_{g'}} + \frac{y_{g'}^2}{2} = 59.17.$$

Neglecting the small hydrostatic term ($y_{g'}^2/2$) for a first approximation:

$$\frac{91.74}{y_{g'}} \approx 59.17 \quad \Rightarrow \quad y_{g'} \approx 1.55 \text{ m}.$$

Refining by trial and error: If $y_{g'} = 1.58$ m: $F_s = \frac{91.74}{1.58} + \frac{1.58^2}{2} = 58.06 + 1.25 = 59.31$ m^2 (Close enough). The required specific force balance is achieved at $y_{g'} \approx 1.58$ m. The new gate opening is:

$$a_{new} = \frac{y_{g'}}{C_c} = \frac{1.58}{0.6} \approx 2.63 \text{ m}.$$

The gate must be closed from 3.33 m to 2.63 m to push the drown jump back to the position immediately downstream of the gate.

5. Effect of Low Tailwater ($y_d = 6$ m)

Swept jump analysis: Comparing the specific forces again:

$$F_s(y_d = 6) = \frac{30^2}{9.81 \times 6} + \frac{6^2}{2} = 15.29 + 18 = 33.29 \text{ m}^2.$$

We know $F_s(y_g = 2) = 47.87$ m^2. Since $F_s(y_g) > F_s(y_d)$, the incoming jet dominates. The jump is pushed far downstream ("swept jump"). The flow downstream of the gate is supercritical. Because the channel is mild, the flow develops an **M3 profile**, gradually increasing its depth (and decreasing its specific force) as it moves downstream. The jump will eventually form at a location x where the supercritical depth $y(x)$ has thickened enough such that $F_s(y(x)) = F_s(y_d)$.

Hazard: In real engineering applications, like energy dissipators, this displacement is problematic because the jump may occur outside the protected stilling basin, causing erosion and potential structural damage to the unlined channel bed.

Retrieving the jump: To bring the jump back to the toe of the gate, we must reduce the specific force of the jet to match the tailwater (33.29 m^2). Since F_s decreases as depth increases (on the supercritical branch), we need to *increase* the gate opening. We seek $y_{g'}$ such that:

$$\frac{30^2}{9.81\, y_{g'}} + \frac{y_{g'}^2}{2} = 33.29.$$

Solving this equation iteratively yields $y_{g'} \approx 3.29$ m.

The required depth is $y_{g'} = 3.29$ m.

$$a_{new} = \frac{3.29}{0.6} \approx 5.48 \text{ m}.$$

The gate must be opened to 5.48 m to reduce the jet velocity and specific force sufficiently to match the low tailwater.

5. Final Comment: Interaction with Downstream Controls

In the previous examples, we simplified the problem by assuming a fixed tailwater depth y_{d}. However, in real hydraulic systems, the downstream depth is rarely a constant value fixed a priori. Instead, it is usually determined by a downstream control structure (such as a weir, a drop, or another gate) or by the channel friction over a long distance.

This downstream condition imposes a subcritical flow profile that propagates upstream towards the gate. In mild slope channels, this resulting backwater curve is typically an **M1 profile** (depth increasing in the flow direction) or an M2 profile (depth decreasing).

To accurately determine the location of the hydraulic jump in such a general scenario, one must solve the gradually varied flow equations for both flow regimes:

1. **Tailwater Profile:** Compute the subcritical water surface profile $y_{\text{sub}}(x)$ starting from the known downstream boundary condition and integrating upstream.
2. **Supercritical Profile:** Compute the supercritical water surface profile $y_{\text{sup}}(x)$ (typically an M3 curve) starting from the vena contracta at the gate and integrating downstream.
3. **Conjugate Profile:** Calculate the conjugate depth profile $y_{\text{conj}}(x)$ corresponding to the supercritical depths $y_{\text{sup}}(x)$ at each location.

The hydraulic jump will form at the specific location x_{jump} where the subcritical tailwater profile intersects the conjugate depth profile of the incoming jet:

$$y_{\text{sub}}(x_{\text{jump}}) = y_{\text{conj}}(x_{\text{jump}}).$$

This intersection point represents the only location where the specific force of the incoming supercritical flow exactly balances the specific force of the downstream subcritical flow, satisfying the momentum principle required for a stationary jump.

Summary

For a sluice gate with discharge per unit width q and fixed tailwater depth y_{d}:

- If $a < a_{\text{l}im}$, then $F_{s,\text{g}}(a) > F_{s,\text{d}}$ and $y_{\text{d}} < y_2(y_{\text{g}})$. The hydraulic jump is free and located downstream from the gate.
- If $a = a_{\text{l}im}$, then $F_{s,\text{g}}(a) = F_{s,\text{d}}$ and $y_{\text{d}} = y_2(y_{\text{g}})$. A stationary free jump forms immediately downstream of the gate.
- If $a > a_{\text{l}im}$, then $F_{s,\text{g}}(a) < F_{s,\text{d}}$ and $y_{\text{d}} > y_2(y_{\text{g}})$. The jump is drowned and shifts upstream.

The system always adjusts the jump position so that the specific force of the supercritical flow feeding the jump matches the specific force of the imposed tailwater depth.